

Study of Residence time distribution for Laminar Flow Reactor with Pulse and Step Inputs to understand difference between theoretical and actual flow of atoms in a Laminar Flow Reactor

Residence Time Distribution Study for LFR

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Residence Time Distribution

Objective:

To measure the residence time distribution with Pulse Input for Continuous Stirred Tank Reactor (CSTR) and Step Input for Laminar Flow Reactor (LFR).

Concept:

In this experiment, the concept and significance of residence time distribution (RTD) was studied and RTD for a given configuration of reactors was estimated. The RTD for a continuous stirred tank reactor (CSTR) and a laminar flow reactor (LFR) was measured as part of this experiment.

In case of a Batch Reactor (BR) all atoms of a reactants and inerts spend exactly the same time in the reactor. In an ideal plug flow reactor (PFR) where axial mixing is absent, all atoms of the reactants spend an equal amount of time in the reactor. Therefore, for a BR or a PFR we have a single value of residence time. However, in an ideal CSTR, not all atoms of the reactants and inerts spend the same time in the reactor and therefore we get a distribution of residence times.

Also, real CSTRs or PFRs may have (i) channeling (bypassing or short-circuiting), which may lead to shorter residence time for the reactant atoms compared to the residence time in an ideal reactor or (ii) dead zones or recirculation zones, leading to trapping of reactant atoms which therefore have a residence time longer than the ideal residence time of ideal reactor. To account for this non-ideality, the residence time distribution or more precisely $E(t)$ function may be looked upon as a correction factor to account for the non-ideality.

The residence time distribution is measured by injecting an inert (non-reactive) tracer into the inlet stream and measuring its concentration in the outlet stream as a function of time. The tracer may be injected all at once to the input stream at a given instance. This is known as pulse input. The tracer may also be injected continuously at a constant rate starting at any given instant. This is known as step input.

Theory:

Pulse input:

Suppose than n number of atoms are injected in a short time.

Since, there is no axial missing in PFR, ideally all these atoms should come out of the reactor at the same time. However, that does not happen in real system. Different time is taken by different atoms to come out of the reactor.

Time spent by atoms in the reactor is called as Residence Time and the distribution of those atoms is called Residence Time Distribution (RTD)

RTD can be experimentally determined after injecting inert chemical called tracer at time $t=0$ and then measuring concentration “ c ” of the tracer in the exit stream as a function of time. We use Pulse and Step input methods of injection.

The area under the C-curve. $E(t)$ is known as the residence time distribution function. Experimentally, the RTD function may be evaluated as:

$$E_i(t) = \frac{c_{out,i}}{\sum c_{out,i} \Delta t_i}$$

The first moment of the residence time distribution gives the mean residence time

$$t_{m,exp} = \frac{\sum t_i \cdot c_{out,i} \Delta t_i}{\sum c_{out,i} \Delta t_i}$$

The second moment of the residence time distribution gives the variance of the distribution

$$\sigma^2_{exp} = \frac{\sum t_i^2 \cdot c_{out,i} \Delta t_i}{\sum c_{out,i} \Delta t_i} - t_{m,exp}^2$$

Step input:

Consider that the tracer is injected via the inlet stream to the reactor at a constant rate $v \cdot C_0$, starting at time $t=0$, prior to which no tracer is added. Here v is the volumetric flow rate and C_0 is the concentration of the tracer in inlet stream. Let $C_{out}(t)$ be the concentration of the tracer in effluent stream at time t . By definition of residence time distribution function, the fraction of tracer that resides in the reactor for time t' is given by

$$E(t') = \frac{v \cdot C_{out}(t')}{\int_0^\infty v \cdot C_{out}(t') dt'} = \frac{C_{out}(t')}{\int_0^\infty C_{out}(t') dt'}$$

If $v \cdot C_0(t-t')dt'$ is the amount of tracer in the inlet stream that enters the reactor in the time interval $t-t'$ and $t-t'+dt'$, where $(0 < t' < t)$, then the amount of tracer injected in this interval that is present in the effluent at time t is given by $v \cdot C_0(t-t') \cdot E(t')$.

The total amount of tracer in effluent at time t is given by

$$v \cdot C_{out}(t) = \int_0^t v \cdot C_0(t-t') \cdot E(t') dt'$$

Since v and C_0 do not vary with time after $t=0$, the concentration of tracer in outlet at time t is

$$C_{out}(t) = C_0 \int_0^t E(t') dt'$$

Therefore,

$$\frac{C_{out}(t)}{C_0} = \int_0^t E(t') dt' = F(t)$$

$F(t)$ is the cumulative residence time distribution function and gives the fraction of the tracer spending time less than t in the reactor.

$$E(t) = \frac{d[F(t)]}{dt}$$

Experimentally, the RTD function may be evaluated as:

$$F_i(t) = \frac{c_{out,i}(t)}{c_0}$$

Continuous Stirred Tank Reactor (CSTR):

Consider a pulse of tracer added to an ideal CSTR at time $t=0$ such that the initial concentration in the reactor is C_0 . No further tracer is added after this. The tracer concentration in the effluent at time t is $C_{out}(t)$, which is the same as the concentration of tracer in the CSTR at time t . The balance on the tracer species is

$$V \frac{d[C_{out}(t)]}{dt} = -v \cdot [C_{out}(t)]$$

Using initial conditions as $C_{out(0)} = C_0$ and substituting

$$\frac{V}{v} = \tau,$$

the space time to integrate, we get

$$C(t) = C_0 \exp\left(\frac{-t}{\tau}\right)$$

The E-curve for CSTR is given by

$$E_{theo}(t) = \frac{C_{out}(t)}{\int_0^\infty C_{out}(t) dt} = \frac{C_0 \exp\left(\frac{-t}{\tau}\right)}{\int_0^\infty C_0 \exp\left(\frac{-t}{\tau}\right) dt} = \frac{1}{\tau} \exp\left(\frac{-t}{\tau}\right)$$

The F-curve for CSTR is given by

$$F_{theo}(t) = \int_0^t E(t') dt' = \int_0^t \frac{1}{\tau} \exp\left(\frac{-t'}{\tau}\right) dt' = 1 - \exp\left(\frac{-t}{\tau}\right)$$

Laminar Flow Reactor (LFR): Consider the LFR as a cylindrical pipe of internal radius R with a parabolic velocity flow

$$u(r) = u_{max} \left[1 - \left(\frac{r}{R}\right)^2\right] = 2 \cdot u_{avg} \left[1 - \left(\frac{r}{R}\right)^2\right] = 2 \cdot \frac{v_0}{\pi R^2} \left[1 - \left(\frac{r}{R}\right)^2\right]$$

Here u_{max} and u_{avg} are the maximum and average flow velocities while v_0 is the volumetric flow rate. If L is the length of the reactor then the residence time for fluid at a radial position r is

$$t(r) = \frac{L}{u(r)} = \frac{\pi R^2 L}{2 \cdot v_0 \left[1 - \left(\frac{r}{R}\right)^2\right]} = \frac{\frac{V}{v_0}}{2 \cdot \left[1 - \left(\frac{r}{R}\right)^2\right]} = \frac{\tau}{2 \cdot \left[1 - \left(\frac{r}{R}\right)^2\right]}$$

Differentiating the above and substituting for $t(r)$ we get the E-curve for the LFR is

$$E_{theo}(t) = \begin{cases} 0, & t < \frac{\tau}{2} \\ \frac{\tau^2}{2 \cdot t^3}, & t \geq \frac{\tau}{2} \end{cases}$$

and, the F-curve for the LFR is

$$F_{theo}(t) = \begin{cases} 0, & t < \frac{\tau}{2} \\ 1 - \frac{\tau^2}{4 \cdot t^2}, & t \geq \frac{\tau}{2} \end{cases}$$

Experimental:
RTD-I:

The system comprises of a laminar flow reactor (LFR) having a volume of 220 ml.

- We have two glass containers, the larger one contains water and the smaller of the two contains the tracer which in our case is 0.012N potassium chloride (KCl) solution
- These two liquids are pumped to the reactors using peristaltic pumps. The one on top is for water and the one at the bottom is for the tracer solution.

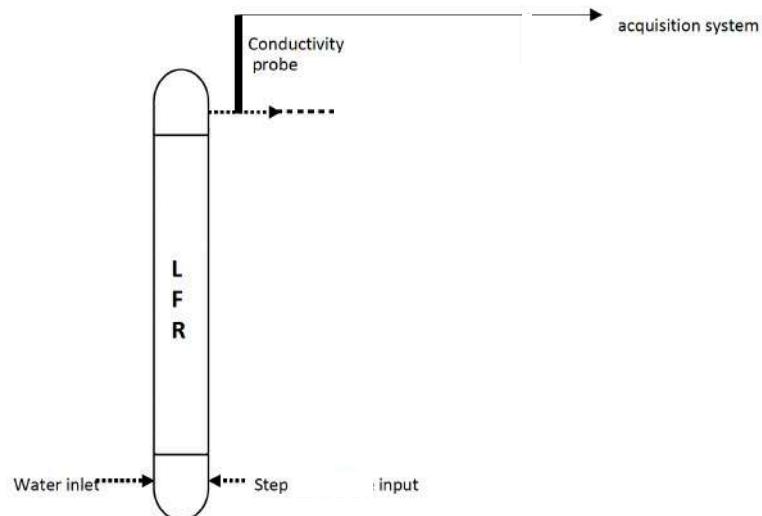


Figure 1. Experimental Setup Residence time distribution

- The tubing from the containers pass through the peristaltic pumps and are connected to the two side inlets at the bottom of the laminar flow reactor.
- The effluent from the laminar flow reactor is discharged into a small container where a probe is used to measure conductivity of the effluent. The conductivity is displayed real time at the top.
- The effluent from the laminar flow reactor forms the feed stream for the continuous stirred tank reactor where the stirrer is driven by the motor shown here.

RTD-II: The system comprises of only the CSTR. Only the pulse input experiment is carried out for this case.

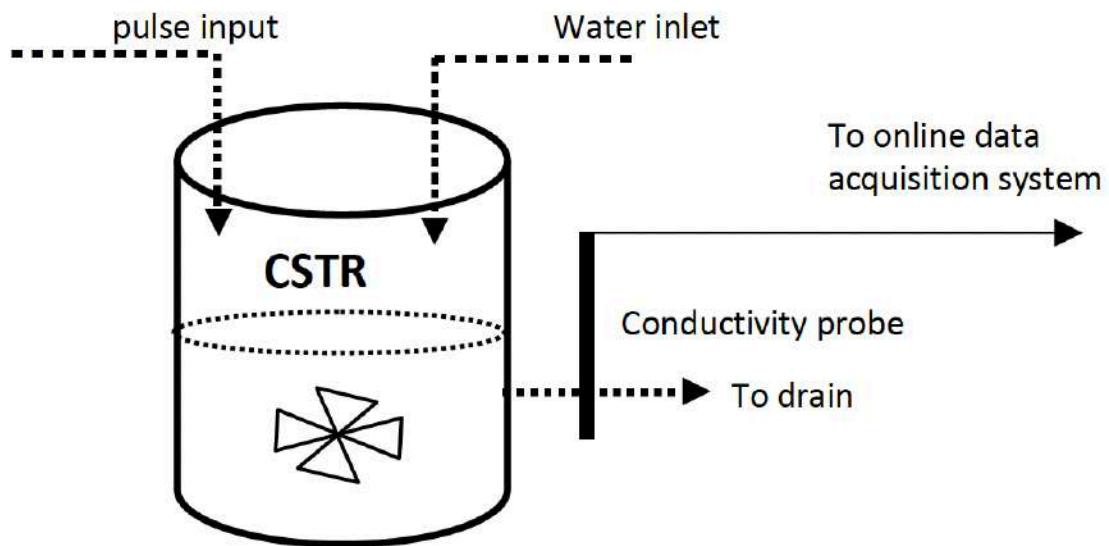


Figure 2: Experimental Setup: Residence time distribution

Procedure:

Pulse Input Experiment: A separate peristaltic pump is used here for the tracer. The tracer for the pulse flow experiment is a 0.2 N solution of potassium chloride which is kept in another container.

- Check whether both the reservoirs (water and tracer) are full
- Prime the tracer feed line to the CSTR is to remove any air in the tubes
- Rinse the CSTR manually by pouring in water from the top and draining the reactor at the base. This ensures that no tracer remains in the reactor before starting the experiment.
- Switch on the motor connected to the stirrer.
- Open the water feed line to the CSTR and fill both reactors with water. Adjust the water flow rate to 60ml/min.
- Open the tracer line to the laminar flow reactor and dispense 20 ml of 0.2 N KCl tracer solution into the reactor at maximum flow rate.
- Then close the tracer inlet line to the reactor. This instant is time t=0

- Record the conductivity values at the exit of the two reactors at 10 sec intervals. Continue to record the conductivity readings till they drop down to near initial values and do not change any further.

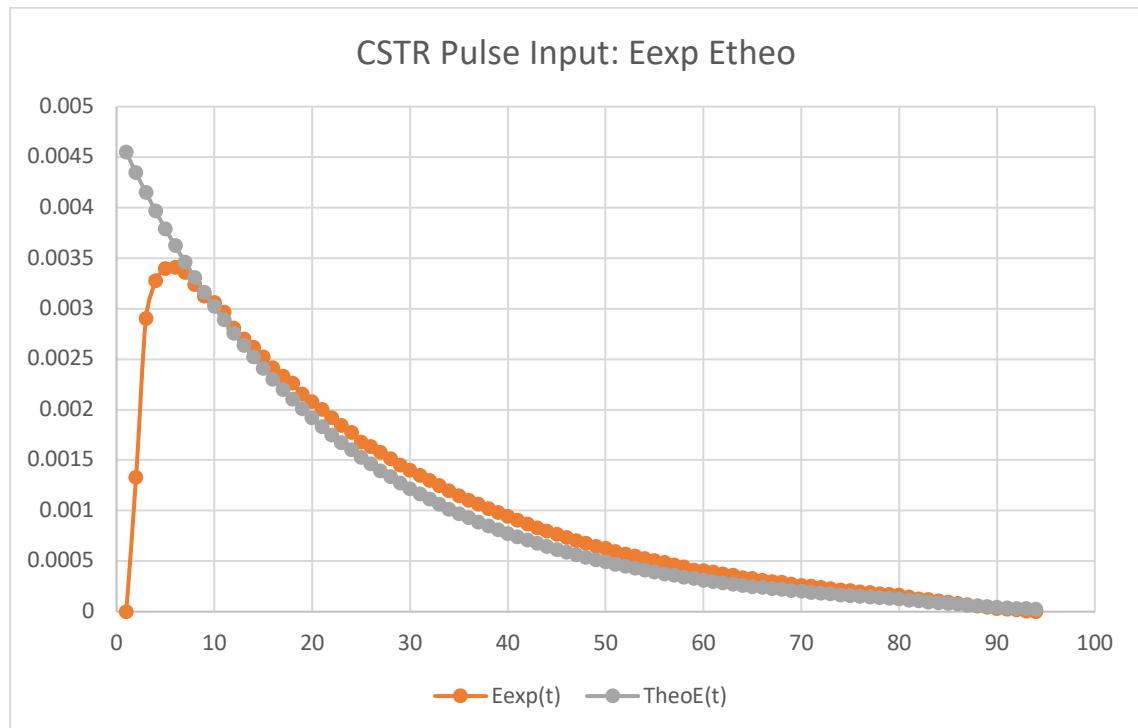
Step Input Experiment: For the step input experiment change the tracer input, container line and pump as the tracer concentration in this case is 0.012N KCl.

- Fill the reactor by perfusing water from the container to the inlet of the laminar flow reactor using the peristaltic pump
- At time $t=0$, the water flow is stopped, and the tracer is perfused through the system at 60 ml/min
- The conductivity readings at the exit of both reactors are to be noted at 10 sec intervals. We see the values increase and then become constant.
- For step input study, start the flow of water through the reactor and allow steady state to be attained.
- Stop recording the conductivity values only after they do not change with time anymore.
- Stop the flow to the reactors and close the inlet valve.
- Switch off the motor connected to the stirrer and drain both the reactors by opening the valves at the bottom.

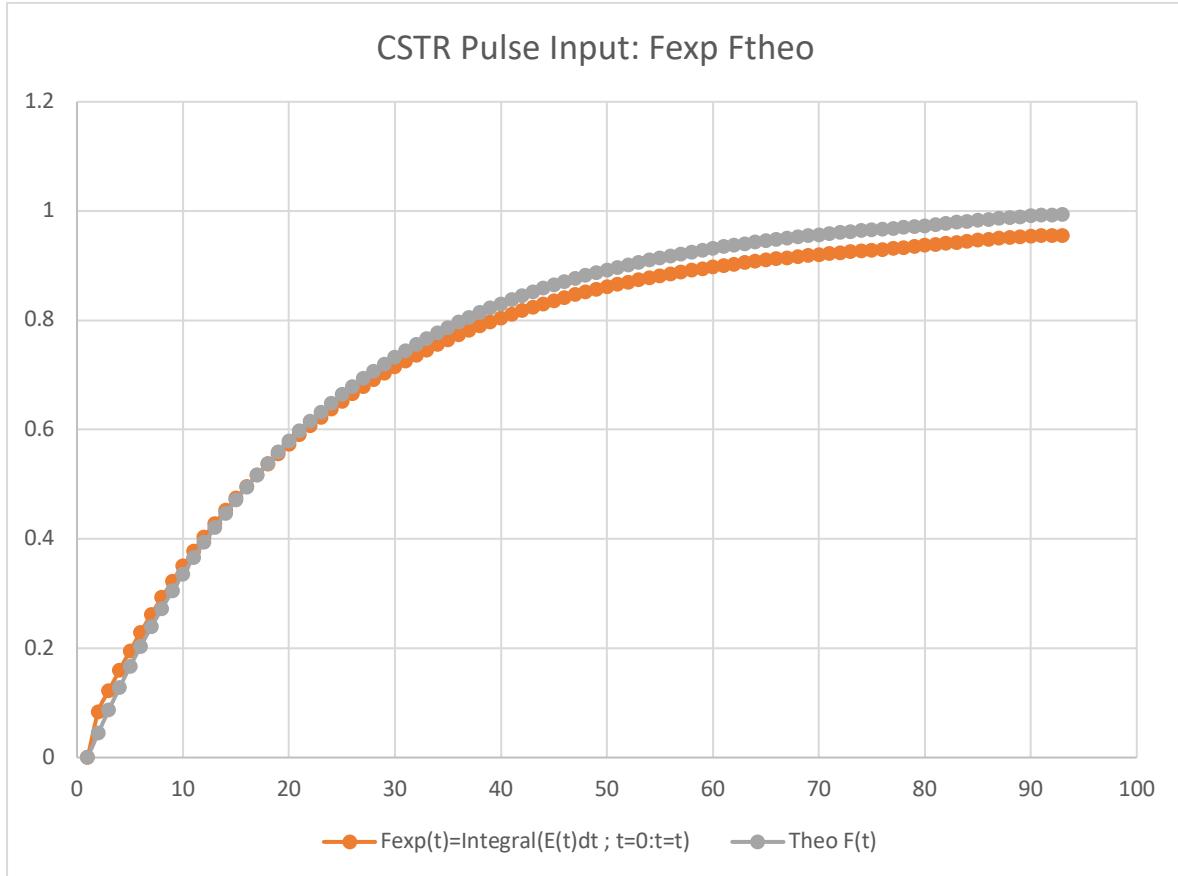
Calculations:

Pulse Input

- Plot $E_{exp,i}(t)$ and $E_{theo}(t)$ vs. t for CSTR to compare
Calculations as per sheet attached

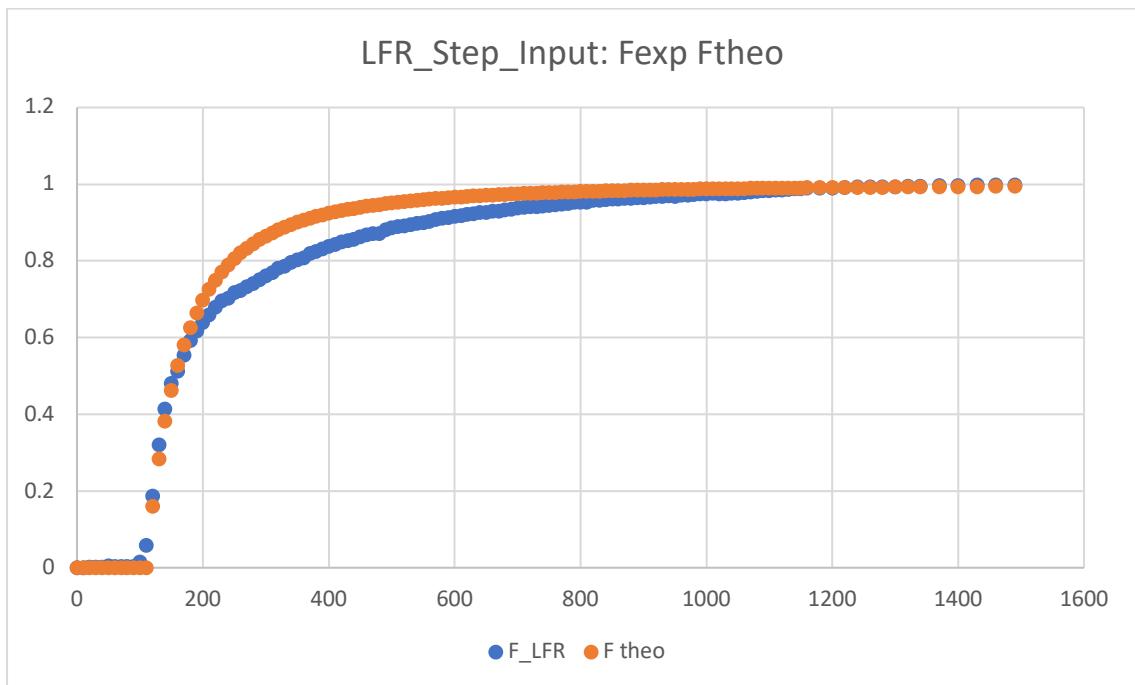


- Plot $F_{exp,i}(t)$ for CSTR and plot $F_{theo}(t)$ for CSTR to compare Calculations as per sheet attached

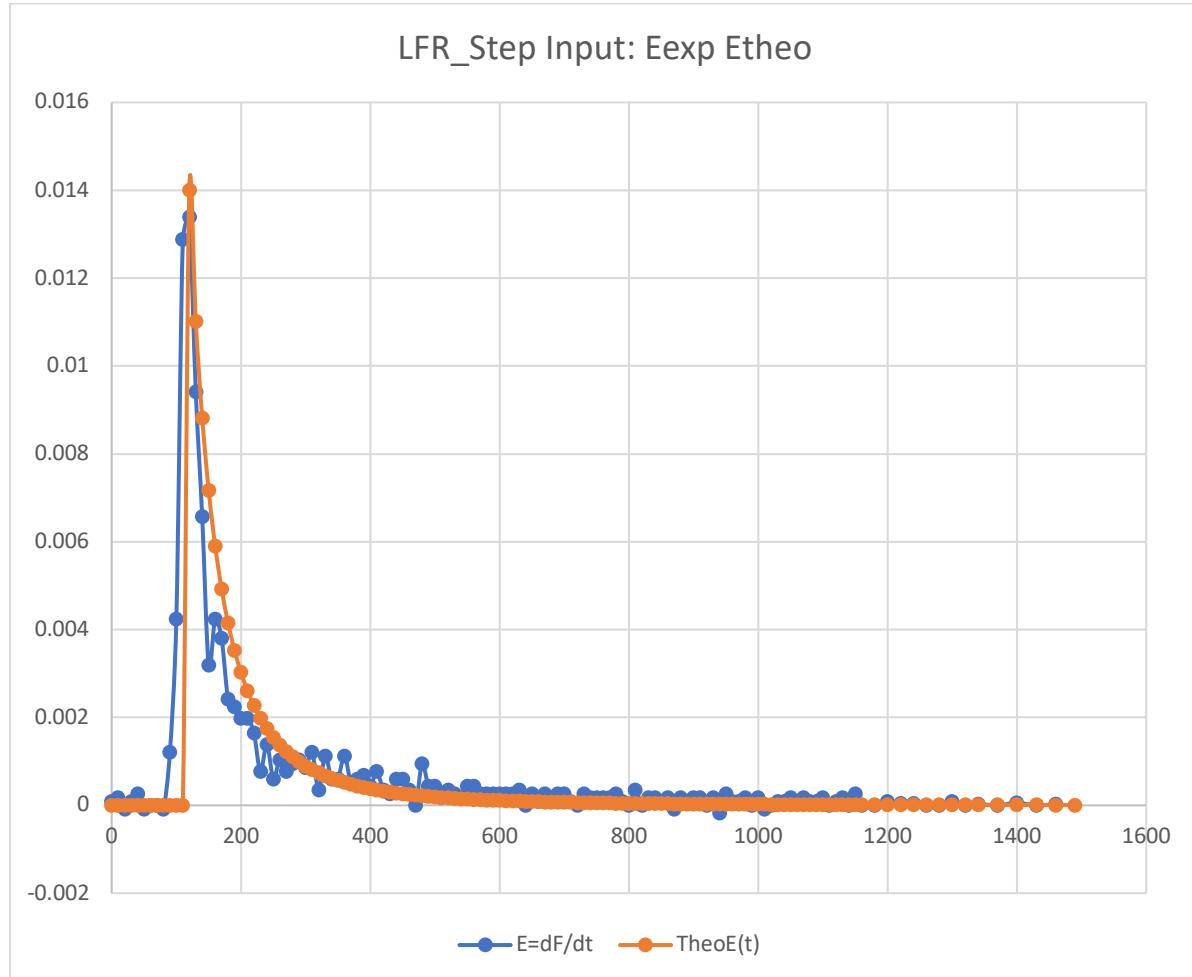


Step Input

- Plot $E_{exp,i}(t)$ and $E_{theo}(t)$ vs. t for LFR to compare - Calculations as per sheet attached



- Plot $F_{exp,i}(t)$ for LFR and plot $F_{theo}(t)$ for LFR to compare - Calculations as per sheet attached



Results and Discussion:

The experiment involved calculations and determination of residence time distribution functions for a pulse input for CSTR and step input in a LFR and following results were obtained

$$\tau = 220$$

In case of Pulse input the conductivity (measure of concentration) rises sharply and declines gradually for CSTR. This is due to constant mixing in CSTR which smoothes out the gradients.

Similar trends are also observed for $E(t)$ distribution functions as well.

For a step input the conductivity (measure of concentration) rises to eventually reach a constant value. The same is observed for LFR. Trends in $F(t)$ are also similar in nature.

The plot of E_{exp} shows a very disturbed one but broad trend curve follows the slope of $F(t)$ curve.

Precautions:

- Ensure that experimental set up is washed properly with distilled water
- Reactor to be properly stirred and the stirring to be continuously maintained
- Ensure proper valve opening before starting the experiment
- Values to be noted in smaller time intervals to get curve
- Overflowing of LFR to be avoided by maintaining proper fluid path and rate

Sources of error

- There can be error in actual deploy and pumped volume of KCl
- Proper steady state is not achieved
- Set up may not be properly cleaned
- Use of tap water instead of distilled water may introduce some errors